Period 4, May 16, 2025.

21. $\cos^2\theta - \sin^2\theta = \sin\theta$

$\cos 2\theta = \sin\theta$

$1 - 2\sin^2\theta = \sin\theta$

$+2\sin^2\theta \quad +2\sin^2\theta - 1$

$0 = 2\sin^2\theta + \sin\theta - 1 \Rightarrow 0 = 2\sin^2\theta + 2\sin\theta - \sin\theta - 1$

$2 \cdot -1 = -2$

$-2+1 = -1$

$2+-1 = 1$

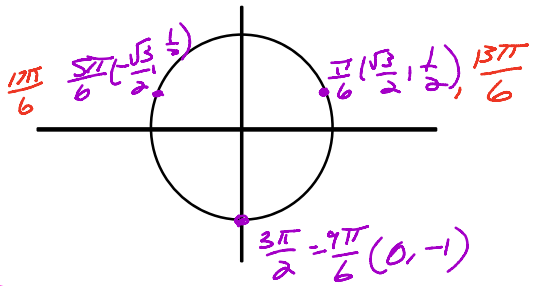
$2\sin\theta(\sin\theta+1) - 1(\sin\theta+1) = 0$

$(\sin\theta+1)(2\sin\theta-1) = 0$

$\sin\theta = -1$ or $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$+\frac{4\pi}{6}$



25. $\sin 2\theta - \sin\theta = 0$

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ or $\cos\theta = \frac{1}{2}$

$\theta = k\pi = n\pi$

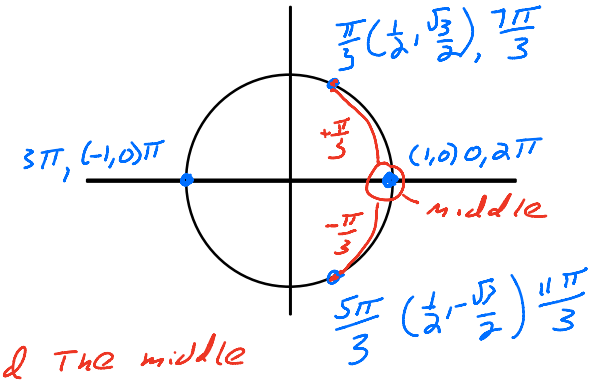
$0, \pi, 2\pi, 3\pi \ldots$

$\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\frac{4\pi}{3} \quad \frac{2\pi}{3} \quad \frac{4\pi}{3}$ ⇒ Find the middle

$0, 2\pi, 4\pi \ldots$

$\theta = 2\pi n \pm \frac{\pi}{3}$



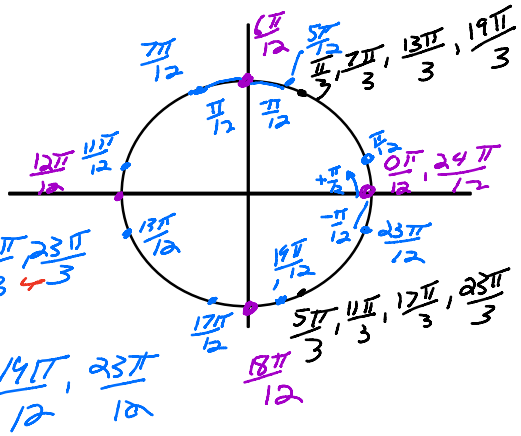
27. $2\cos^2 2\theta - 2\sin^2 2\theta = 1$

$\cos^2 a - \sin^2 a = \cos 2a$

$2(\cos^2 2\theta - \sin^2 2\theta) = 1$

$\frac{2\cos 4\theta}{2} = \frac{1}{2}$

$\cos 4\theta = \frac{1}{2}$



$4\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \frac{19\pi}{3}, \frac{23\pi}{3}$

$\frac{4\theta}{4} = \frac{\pi}{4\cdot 3}, \frac{5\pi}{4\cdot 3}, \frac{7\pi}{4\cdot 3}, \frac{11\pi}{4\cdot 3}, \frac{13\pi}{4\cdot 3}, \frac{17\pi}{4\cdot 3}, \frac{19\pi}{4\cdot 3}, \frac{23\pi}{4\cdot 3}$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$0, \frac{\pi}{2}, \frac{2\pi}{2}, \frac{3\pi}{2}, \frac{4\pi}{2}$

$\frac{\pi}{2}$ $\frac{\pi}{2}$

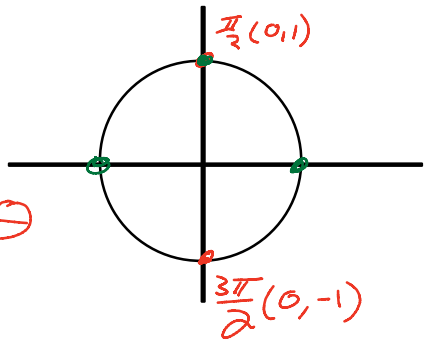
$a_n = 0 + \frac{\pi}{2}(n-1)$

$a_n = \frac{\pi}{2}n - \frac{\pi}{2}$ or $\theta = \frac{\pi}{2}n \pm \frac{\pi}{12}$

$\frac{1 - \sin\theta}{\cos\theta} = \cos\theta \cdot \cos\theta$

$\cos\theta \neq 0$

$\frac{\pi}{2}$ and $\frac{3\pi}{2} \neq \theta$



$1 - \sin\theta = \cos^2\theta$

$1 - \sin\theta = 1 - \sin^2\theta$

$+\sin^2\theta$ $+\sin^2\theta$

$\sin^2\theta - \sin\theta = 0$

$\sin\theta(\sin\theta - 1) = 0$

$\sin\theta = 0$ or ~~$\sin\theta = 1$~~

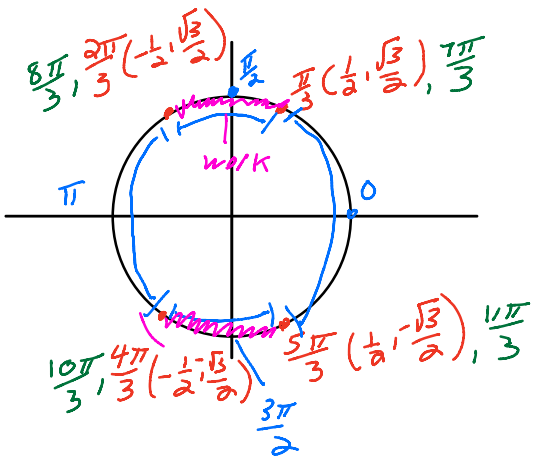
$\cos\theta = 0$ not 1 or 2

$\theta = 0, \pi, 2\pi, 3\pi,$

$\theta = n\pi$ or $k\pi$

$|\cos\theta| \le \frac{1}{2}$

$-\frac{1}{2} \le \cos\theta \le \frac{1}{2}$



Start Finish

$\frac{\pi}{3} \le \theta \le \frac{2\pi}{3}$

$\frac{4\pi}{3} \le \theta \le \frac{5\pi}{3}$

$|a| < b$ b must be Positive

$-b < a < b$

$|a| > b$ b must be positive or

$a > b$ or $a < -b$

$|a| > -0.2$

$a = \mathbb{R}$

Test

$|\cos 0| \le \frac{1}{2} \Rightarrow 1 \le \frac{1}{2}$ False

$|\cos\frac{\pi}{2}| \le \frac{1}{2} \Rightarrow 0 \le \frac{1}{2}$ True

$|\cos\pi| \le \frac{1}{2} \Rightarrow 1 \le \frac{1}{2}$ False

$|\cos\frac{3\pi}{2}| \le \frac{1}{2} \Rightarrow 0 \le \frac{1}{2}$ True

Generalization

Start

$\frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3} \ldots$

$\frac{3\pi}{3}$ $\frac{3\pi}{3}$

$a_n = \frac{\pi}{3} + \frac{3\pi}{3}(n-1)$

$a_n = \frac{3\pi}{3}n - \frac{2\pi}{3}$

End

$\frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}, \frac{11\pi}{3}$

$\frac{3\pi}{3}$

$a_n = \frac{2\pi}{3} + \frac{3\pi}{3}(n-1)$

$a_n = \frac{3\pi}{3}n - \frac{\pi}{3}$

$\pi n - \frac{2\pi}{3} \le \theta \le \pi n - \frac{\pi}{3}$